5 Calculations for Structures under Mechanical Load – Examples of Geometrically Simple Structural Parts under Static Loads

5.1 Specific Materials and Processing Problems

The mechanical properties of polymeric materials, especially those of thermoplastics, depend to a much greater extent on temperature, time, and on the magnitude and nature of an applied load than those of metals. In addition, many environmental effects, such as UV radiation or exposure to certain chemicals, play a significant role in aging and related changes in properties. This can be difficult to quantify in strength-related calculations. The conditions used in processing (e.g., injection molding process) can have an effect on the properties of the finished product.

Because the strength of most polymeric materials is an order of magnitude less than that of metals, components made from these materials may be highly stressed even under relatively low loads. On the other hand, a component made from a polymeric material is more likely to be rendered unusable by a high degree of deformation than by catastrophic failure due to fracture (in the case of ductile polymeric materials). The modulus of elasticity of these materials differs by as much as two orders of magnitude. The complexity of deformation behavior, however, leads to the expectation that deformation can only be calculated precisely with the aid of computer support and that the empirical determination of physical properties for this purpose will be very costly. There are, however, a limited number of design tasks for which such extensive design work is necessary. Not all designers have the extensive knowledge required to optimize structural elements with the aid of FEM. This task is best performed by an engineer who specializes in computer aided finite element structural design. However, there are a large number of applications where routine structural design calculations can be done by the average designer to help determine optimum part geometry. These are carried out using justifiable simplifications [5.2]. Knowledge of some specific properties of polymers is needed for this purpose.

5.1.1 Deformation Behavior under Uniaxial Dynamic Tensile Stress

The deformation of polymeric materials under the action of an external force, can be described by three components, which to some extent are superimposed, but at the same time predominate in certain ranges of deformation (see also Section 4.1).

Linear elastic, spontaneously reversible deformation is restricted in most polymeric materials to a total strain range of less than one tenth of one percent. This marks the end of the range of applicability of Hooke's law of elasticity for many polymeric materials. Most strains observed
in practice exceed this range. In many unreinforced thermoplastics, it is not even possible to
demonstrate (by way of stress-strain experiments) the existence of a range in which Hooke’s
law is obeyed.

As strain increases, visco-elastic deformation processes appear. These are said to fall into the
*linear visco-elastic range* when the laws of linear visco-elasticity apply. This range is characterized
in that two strains $\varepsilon_1$ and $\varepsilon_2$ may be added when the corresponding stresses $\sigma_1$ and $\sigma_2$ are
superposed (Boltzmann’s law of superposition).

\[ \begin{align*}
\sigma_1 & \Rightarrow \varepsilon_1 \\
\sigma_2 & \Rightarrow \varepsilon_2 \\
\sigma_1 + \sigma_2 & = \varepsilon_1 + \varepsilon_2
\end{align*} \] (5.1)

In isochronous stress-strain diagrams, this range is marked by the end of the straight-line rise
of the isochrones. This range also ends for most polymeric materials at stresses which cause
strains of 0.5 to 1%. As strain increases further, the relationship of stress to strain no longer
depends only on time (and of course on temperature), but rather on the applied stress itself.
The deformation processes become increasingly nonlinear and also irreversibly (or partially
irreversibly) viscous.

These deformation phenomena found at the macroscopic level are characterized by the
molecular deformation and damage mechanisms occurring in the material (see also Chapter 4).

In some applications, thermoplastic polymeric materials are subjected to loads which take
them into the nonlinear stress-strain range. As a result, calculations based on the laws of
elasticity inevitably yield results which diverge to a greater or lesser extent from actual behavior.
As long as the stresses are at least close to the linear viscoelastic range, the formulae of the
theory of elasticity afford a satisfactory degree of accuracy. Therefore, their use is certainly
justifiable and may indeed be essential, based on time available and on economic grounds.

In addition, the calculation of structural elements made from polymeric materials is rendered
more difficult and uncertain due to the fact that the condition of a homogeneous, isotropic
continuum is not fulfilled. Here, the different types of anisotropy should be taken into account
whenever possible (*i.e.*, when anisotropic data is available). The causes of these may lie in the
material itself (reinforcement by unidimensional fibers, for instance) or be due to processing.
Examples of the latter include residual molecular alignments and internal stresses imposed
during the molding operation (see also Chapters 2 and 7).

Internal stresses can be caused by impediments to shrinkage in macroscopic domains.
A number of primary causes of internal stress may be distinguished. Cooling internal stresses
are the result of different rates of cooling over the cross section of the molding. Holding-
pressure related internal stresses are due to the holding pressure acting in the interior of the
molding in the injection molding process when the outer contours have already solidified.
Embedding internal stresses arise from shrinkage impeded by the shape of the molding due,
for example, to metal inserts or due to the constraints imposed by the shape of the mold itself.
Secondary internal stresses are also known. These include structural internal stresses brought
about by curing reactions (in thermosets) or crystallization (in semi-crystalline thermoplastics).
Embedding internal stresses can be produced, for example, by the incorporation of fillers.
5.2 Determination of Strength

5.2.1 Basic Procedure for Structural Part Design

The basic procedure for structural part design is given in Figure 5.1. An analysis of stress provides information about the magnitude and nature of the stresses at work in the theoretical cross section in question. Multiaxial states of stress are transformed using suitable failure criteria into a uniaxial reference stress having the same effect which is compared with the permissible level of stress. The latter is obtained from a characteristic property value specific to the material being considered for the application (e.g., tensile yield strength etc.). This property value is further reduced by an appropriate safety factor and any applicable reduction factors.

According to this analysis, the fundamental equation for determining strength may be written down as:

\[
\sigma_{v,\text{max}} \leq \sigma_{\text{perm}} = \frac{K}{S \cdot A}
\]  

(5.2)

where

- \(\sigma_{v,\text{max}}\) = maximum stress occurring in section being analyzed
- \(K\) = characteristic strength property of the material
- \(S\) = safety factor
- \(A\) = material-specific reducing factor

Cooling internal stresses give rise to compressive stresses on the surface of the molding. These compressive internal stresses can have a positive effect in the event of tensile loads in the outer zone (e.g., during bending). On the other hand, excessive holding pressure causes compressive internal stresses in the interior of the molding and tensile internal stresses in the outer zone. These are superimposed on the cooling internal stresses so that when the holding pressure is high enough, tensile internal stresses may also be apparent on the surface of the molding. These internal stress states, which are also very difficult to describe quantitatively, represent a further uncertainty factor in calculations of strength.

![Figure 5.1 Basic procedure for structural part design [5.1]](image)
5.2.1.1 Characteristic Strength

A number of different (material specific) strength parameters can be used for structural design, depending on the specific material behavior.

Figure 5.2 shows the most important failure characteristics.

Materials exhibiting largely brittle failure (see Figure 5.2a, corresponding to type A in Table 4.1) or having a distinct yield point (see Figure 5.2b, corresponding to types B and C in Table 4.1) have a clear-cut characteristic value that can be used as a “failure stress”. This is more difficult to define when there are no such prominent features in the stress-strain behavior, as is the case for many thermoplastics, especially at elevated temperatures. In these cases, a stress is proposed that causes a deformation or strain of a certain magnitude (0.5% offset has proved to be useful) in the nonlinear range (see Figure 5.2c, corresponding to type D in Table 4.1). In amorphous (transparent) polymeric materials, craze zones form when a certain damage stress is exceeded. Although in the initial stages these craze zones (see also Figure 4.2) are still capable of bearing loads, macroscopic cracks form in the course of further strain. This craze limit stress (onset of crazing) can be used as the failure value if available (see Figure 5.2d). Knit or weld line strength may be the limiting factor for structural parts containing weld lines (especially for reinforced thermoplastics).

In the case of sustained static loads, the creep strength is selected as the failure value (see Figure 5.3a) or the value $\sigma^*$ in Section 4.2.1.

Under dynamic loads, the characteristic failure value is correspondingly obtained from the Wöhler curve or the Smith diagram (see Figures 5.3b and c). Alternatively, a limiting value as shown in Table 4.3 or a load-cycle-dependent rigidity value (see Figure 4.20) is introduced.

![Figure 5.2](image1.png)  ![Figure 5.3](image2.png)

**Figure 5.2** Some common short-term stress values that may be suitable as “failure stress” values [5.1]

**Figure 5.3** Common dimensioning characteristics for long-term loads
Of course, in determining characteristic failure values, the high temperature-dependence of thermoplastics must be taken into account. Here, heat may arise as an external effect or be generated by friction or damping. This situation generates problems in determining and using characteristic values for dynamic failure. This is especially true if long-term aging is a significant factor.

In many cases (e.g., in snap-fit connections), strain-based structural design is more appropriate than stress-based designs. Design calculations are done on the basis of a limiting strain that must not be exceeded. It should be noted that in cases of nonlinear deformation behavior, there is a difference between operating, for example, at 80% of the yield strain and operating at 80% of the yield stress. Different values are obtained as depicted in Figure 5.4.

In [4.18] Oberbach proposes that permissible stress values be determined using a type of nomograph. The nomograph makes a distinction between short-term, long-term, and dynamic stresses; short-term stress being subdivided into single and multiple loading. A distinction is also made between ductile semi-crystalline, brittle amorphous, and glass-fiber reinforced thermoplastics. The procedure is illustrated in Figure 5.5.

The nature of the load and the group into which the material falls lead to an A-factor on the right ordinate of the left-hand part of the figure (in the example, single loading/glass-fiber reinforced thermoplastic = 0.68). The breaking load (in the example $\sigma_B = 144$ MPa) is multiplied by this factor to obtain the design stress at room temperature. The ordinate of the stress-strain diagram on the right-hand part of the figure is divided into fractions of the

![Figure 5.4](image.png)

**Figure 5.4** A percentage safety interval in a strain-based analysis affords a permissible value different from that in a stress-based analysis for materials that exhibit non-linear stress-strain behavior.
Calculations for Structures under Mechanical Load

[References on Page 211]

Breaking load at 23 °C. From the isotherms a permissible strain value can be determined at the requisite temperature for the design stress determined in this way (in the example $\varepsilon_{\text{perm}} = 1.2\%$). While keeping this strain value constant, the A-factor at any other temperature can now be determined at the point of intersection with the corresponding isotherm. This allows the calculation of the permissible stress at this temperature. The same procedure may be used to determine the effect of time for isochronous stress-strain curves.

5.2.1.2 Safety Factors

The magnitude of the safety factors used in structural design of load bearing plastic parts depends on a number of variables, including the various uncertainties in calculating and determining the characteristic values of a material. Such uncertainties often arise, for example, when a load is first applied and are further complicated by simplified assumptions concerning geometry, stress, or previous processing. Secondly, the magnitude of safety factors depends on the seriousness of the damage which would occur were the structural part to fail. It is also influenced by the specific characteristic failure value against which the theoretical calculation is intended to provide a safeguard. It may be the designer who determines the magnitude of the safety factor in some applications. In some cases, agencies also prescribe minimum levels of safety for load-bearing structural parts.

The designer is responsible for the first group of safety factors identified above. He or she must estimate the effect of the simplifications made and the unquantifiable processing operations and rate them according to importance. For the second category of safety factors, the following guide values apply, provided they have not been previously described or agreed differently.

Figure 5.5 Estimation of permissible stresses and strains for a glass-fiber reinforced PBT (30% by weight)
\[ S_{\text{min}} \geq 2 \text{ for calculations safeguarding against fracture} \]
\[ S_{\text{min}} \geq 3 \text{ for calculations safeguarding against bending and buckling} \]
\[ S_{\text{min}} \geq 1.2 \text{ for calculations safeguarding against fracture stresses due to cracking with the additional condition that } S_{\text{min}} \geq 2 \text{ for strength} \]
\[ S_{\text{min}} = 1.0 \text{ for calculations of } S_{0.5\%}, \text{with the additional condition that } S_{\text{min}} \geq 2 \text{ for strength.} \]

### 5.2.1.3 Reducing Factors

Other uncertainties attributable to the lack of physical properties data under special conditions should not be considered as safety factors but rather be taken into account in the form of what are known as reducing factors in order not to obscure the focus on strength. These material-specific reducing factors have values greater than one.

\[ \sigma_{\text{perm}} = \frac{K}{S} \cdot \frac{1}{A_T} \cdot \frac{1}{A_{\text{st}}} \cdot \frac{1}{A_{\text{dyn}}} \cdot \frac{1}{A_A} \cdot \frac{1}{A_W} \ldots \]  

(5.3)

\[ A_T \text{ takes account of the effect of temperature on the yield stress and/or tensile strength and can be determined between } 0 \, ^\circ\text{C} \text{ and } 100 \, ^\circ\text{C} \text{ by the following relationship when } K \text{ at } 20 \, ^\circ\text{C} \text{ is inserted into equation (5.2).} \]
\[ A_T = \frac{1}{1 - [k(T - 20)]} \]  

(5.4)

Values for \( k \) in this equation are as follows:
- PA 66 = 0.0112
- PA 6 = 0.0125
- PBT = 0.0095
- PA GF and PBT GF = 0.0071
- POM = 0.0082
- ABS = 0.0117

These values were obtained by linear interpolation of the plot of strength or yield stress in the temperature range between 0 and 100 °C. It is of course more sensible to safeguard against the characteristic failure value at the temperature in question, because these values are often available. On the other hand, Eq. 5.4 can also be employed to provide a rough estimate of the effect of temperature on other characteristic mechanical properties, if no experimental data are available at the required temperature. Testing does provide another option.

\( A_{\text{st}} \) takes account of the duration of a static load and can be substituted by the values of:
- 1.3 for a loading time of a few hours
- 1.6 for a loading duration measured in weeks
- 1.7 for a loading duration measured in months
- 2.0 for a loading period of a few years.
$A_{\text{dyn}}$ takes account of the effect of dynamic loading and may be taken to be approximately 1.3 to 1.6.

$A_A$ can also cover any aging effects (see Section 4.6.3).

Properties of materials which undergo change due to the absorption of water must be reduced by the reducing factor $A_W$. For unreinforced polyamides, this can be obtained starting from the value for strength in the dry state from:

$$A_W = \frac{1}{1 - 0.22w}$$  \hspace{1cm} (5.5)

where $w$ is the moisture content in percent by weight assuming uniform distribution over the cross section. This applies within the limits 0% < $w$ < 3%. Above a moisture content of 3% by weight $A_W = 3.4$, likewise starting from strength in the dry state.

Under other conditions (e.g., chemical exposure etc.), it may be necessary to consider other reducing factors.

The process of differentiation should cause the designer to ponder deeply about any possible effects which may reduce strength. Discussions with other members of the design team can be helpful here.

### 5.2.2 Uniaxial State of Stress

The general equation of stress for uniaxial tensile loads having uniform distribution of stress is as follows:

$$\sigma = \frac{F}{A}$$  \hspace{1cm} (5.6)

Another uniaxial, but inhomogeneous, state of stress occurs in the case of bending, e.g., when a beam or similar cross section has a bending moment applied at its ends.

Due to the transverse forces produced by bending loads, nonuniformly distributed shear stresses additionally appear over the cross section of the beam. These shear stresses originating from cross-force bending are, however, negligible when $l/h \geq 1$. Accordingly, pure bending may be assumed in the design of spiral springs and elastic hooks (snap-fit connections), as long as they fulfill the aforementioned condition. In practical applications, these structural parts are likewise frequently loaded beyond the linear viscoelastic range. It has, however, proved effective to use the equation for the elastic case even in such cases and in this way to determine a theoretical outer fiber stress. The values determined using elastic theory are greater than the actual stresses. When quantified, the non-linear behavior can be taken into account for a more realistic prediction.
5.2.2.1 Example of a Thin-Walled Pipe under Internal Pressure

It is extremely rare for components to be in a simple, uniaxial state of stress, although it is sometimes the case. An example is provided by a thin-walled pipe under internal pressure when only the mean tangential stress is considered. This may be regarded in approximation as a thin-walled hub mounted on a metal bolt. Under constant pressure, the stress in the interior of the pipe remains constant and the permissible maximum stress to avoid bursting of the pipe is given by:

\[ \sigma_{\text{perm}} = \frac{\sigma_{B}(T, t)}{S \cdot A} \]  

(5.7)

where \( \sigma_{B} \) for the bursting failure is taken from the stress-strain diagram or, in the case of creep failure, from the creep diagram. In Table 5.1 some results from bursting tests on thin-walled pipes are presented together with the tangential stress calculated using what is known as the “boiler formula”

\[ \sigma_{t} = \rho \frac{r}{s} \]  

(5.8)

using the tensile strength measured in tensile tests.

It may be seen that this comparison reveals very good agreement for a variety of different thermoplastic materials.

Table 5.1 Comparison of Calculated Burst Stress for Thin-Walled Pipes and Measured Tensile Strength Values for Several Thermoplastics

<table>
<thead>
<tr>
<th>Material</th>
<th>Wall thickness s [mm]</th>
<th>Mean radius ( r_{m} ) [mm]</th>
<th>Bursting pressure ( p ) [bar]</th>
<th>Stress in pipe ( \sigma_{t} ) [MPa]</th>
<th>Tensile strength ( \sigma_{a} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDPE (5261 Z)</td>
<td>1.5</td>
<td>5.0</td>
<td>85</td>
<td>28.3</td>
<td>28</td>
</tr>
<tr>
<td>HDPE (5261 Z)</td>
<td>1.0</td>
<td>4.5</td>
<td>64</td>
<td>28.8</td>
<td>28</td>
</tr>
<tr>
<td>HDPE (5261 Z)</td>
<td>0.75</td>
<td>4.25</td>
<td>50</td>
<td>28.3</td>
<td>28</td>
</tr>
<tr>
<td>PVC</td>
<td>0.75</td>
<td>5.25</td>
<td>91</td>
<td>63.7</td>
<td>58</td>
</tr>
<tr>
<td>PVC</td>
<td>0.75</td>
<td>5.25</td>
<td>89</td>
<td>62.3</td>
<td>58</td>
</tr>
<tr>
<td>PMMA (G 55)</td>
<td>0.75</td>
<td>6.25</td>
<td>72</td>
<td>60.0</td>
<td>62</td>
</tr>
<tr>
<td>PMMA (G 55)</td>
<td>0.75</td>
<td>4.75</td>
<td>96</td>
<td>60.8</td>
<td>62</td>
</tr>
</tbody>
</table>
Example

What is the bursting pressure of a pipe made of POM GF20 at RT for which
\( s = 1.5 \text{ mm} \)
\( r_m = 5.0 \text{ mm} \)
and what is the approximate increase in diameter when this happens?
From CAMPUS, single-point data, Ticona, for Hostaform C 9221 GV1/20:
\( \sigma_B = 105 \text{ MPa} \)
\( \varepsilon_B = 2.5\% \)
\[
\begin{align*}
p &= \frac{\sigma_B \cdot s}{r_m} = \frac{105 \cdot 1.5}{5} = 31.5 \text{ N/mm}^2 = 315 \text{ bar} \\
\Delta d &= d \cdot \varepsilon = \frac{5 \cdot 2.5}{100} = 0.125 \text{ mm}
\end{align*}
\]
The pipe ruptures at approximately 315 bar and the increase in diameter is approximately 0.125 mm.

5.2.3 Multiaxial State of Stress

Most engineering components are subjected to a multiaxial state of stress due to the external forces acting on them. The question whether this state of stress will result in failure is answered with the help of a suitable failure criterion. Using mathematical formulae for all possible states of stress resulting in failure, a multiaxial state of stress is converted to a theoretical, uniaxial resultant load, which acts on the material in a manner comparable with the actual multiaxial load and may cause failure. As a general rule, this theoretical load is taken as the characteristic tensile failure value. Failure criteria can be developed on the basis both of physical models of the failure event as well as mathematical approaches derived from empirical observations.

Because the effort involved in verifying test data experimentally is considerable, there are few confirmed results to verify that this approach can be used reliably to design polymeric material structures subjected to multiaxial stresses. References [5.3] and [5.4] provide an overview of currently known criteria.

5.2.3.1 Failure Criteria

"Classical" engineering failure criteria do not describe the behavior of polymeric materials with sufficient accuracy. Nevertheless, the shear experiments discussed in Section 5.2.3.2 show, for example, that some states of stress involving only shear may be described with sufficient apparent accuracy by Tresca’s simple mathematical shear stress criterion:
\[
\sigma_v = \sigma_1 - \sigma_3 \quad (5.9)
\]
8 Flexing Elements

Structural elements that are required to have high deformability should be designed so that they are capable of withstanding the flexural or torsional loads associated with the application (see also Section 6.1). Two examples of such designs common in parts made from polymeric materials are snap-fit or interlocking joint elements and elastic elements. Another common feature in parts designed for high deformability is their relatively thin wall thickness. For example, integral hinges are structural elements having extremely low wall thicknesses.

8.1 Snap-Fit Joints

Definition

Joint types are defined according to the mechanisms acting at the points of attachment holding the assembled parts together (see Figure 8.1) [8.1]. On this basis, a snap-fit joint is a frictional, form-fitting joint.

The structural features of a snap-fit joint are hooks, knobs, protrusions, or bulges on one of the parts to be joined, which after assembly engage in corresponding depressions (undercuts), detents, or openings in the other part to be joined.

Accordingly, the design of a snap-fit joint is highly dependent on the polymeric material(s). Snap-fit joints are also relatively easy to assemble and disassemble. A key feature of snap-fit joints is that the snap-fit elements are integral constituents of the parts to be joined.

Figure 8.1 Types of joints (schematic) [8.1]
a) Form-fitting joint  
b) Frictional joint  
c) Adhesive joint  
d) Frictional form-fitting joint

←→ Direction of action of forces
**Differentiated and Integrated Construction**

Design solutions using “differentiated” construction assign certain functions separately to the individual structural elements with the goal of fulfilling all of the functional requirements in an optimum manner. This inevitably means that there are a number of parts in a subassembly. “Integrated” construction, on the other hand, uses fewer parts and consequently results in lower assembly costs but may require the acceptance of restrictions or compromises in functionality. Figure 8.2 shows this trade-off with reference to the example of a bayonet coupling.

![Figure 8.2](image)

**Figure 8.2** Design variants for a coupling as described in [8.12] and [8.16]

The systematic reduction in the numbers of parts finally leads to variant d), a snap-fit joint made from polymeric material. Injection molding technology is so versatile, that it allows for the integration of functions directly into the parts to be joined.

**Classification**

Snap-fit joints are classified according to the most varied attributes [8.2, 8.3, 8.4, 8.13]. However, a classification based on geometrical considerations appears to be most appropriate here (see Figure 8.3).

![Figure 8.3](image)

**Figure 8.3** Classification scheme for snap-fit elements based on geometrical considerations
Dimensions and Forces

The dimensions and forces associated with assembly/disassembly are discussed in the following figures.

Figure 8.4 Dimensions and their designations for snap-fit hooks
- $\alpha_1$ = Joining angle
- $\alpha_2$ = Retaining angle
- $b$ = Breadth of cross section (hook breadth)
- $h$ = Height of cross section
- $l$ = Snap-fit length
- $H$ = Snap-fit height (undercut)

Figure 8.5 Dimensions and their designations in cylindrical annular snap-fit joints
- $d_{\text{max}}$ = Greatest diameter
- $d_{\text{min}}$ = Smallest diameter
- $d_o$ = Outer diameter
- $s_o$ = Wall thickness
- $d_i$ = Inner diameter
- $s_i$ = Wall thickness

Figure 8.6 Dimensions and their designations in spherical annular snap-fit joints

Figure 8.7 Dimensions and their designations in torsional snap-fit joints
- $l_T$ = Length
- $r_T$ = Radius
- $\beta$ = Torsion angle
- $\gamma$ = Twisting angle
- $l_{1,2}$ = Lever arm lengths
- $f_{1,2}$ = Elastic excursions
- $Q_{1,2}$ = Deflection forces
The forces and angles at the assembly contact surfaces of the joints (see Figure 8.8) apply in an analogous manner for all snap-fit joint design variations.

**Assembly Operation**

A review of the snap-fit assembly operation is helpful to gain a better understanding of the factors at work and of the calculations discussed below. The assembly force \( F \), generally acting in the axial direction, is resolved at the mating surface in accordance with the mathematical relationships associated with a wedge (see Figure 8.8). The transverse force \( Q \) causes the deflection needed for assembling the joint. At the same time, friction and the joining angle determine the conversion factor \( \eta \).

\[
\eta = \tan(\alpha_1 + \rho) = \frac{f + \tan \alpha_1}{1 - f \cdot \tan \alpha_1} \tag{8.1}
\]

The relationship in Eq. 8.1 is plotted in Figure 8.9 against \( \alpha_{1,2} \) for common values of \( \eta \).

The retaining or release force of the joint can be altered using the retaining angle \( \alpha_2 \). The use of a value of \( \alpha_2 \geq 90^\circ \) creates a self-locking geometric form-fitting joint. Figure 8.10 illustrates that a joint constructed in this way can be released again without forced failure of the joint when the moment of the force couple represented by the retaining and reaction forces is able to overcome the friction force in the active surface.

A design countermeasure to prevent release in this way is to attach a retaining guard or locking ring (see also Sections 8.1.1.3 and 8.1.3.3).

As snap-fit features are being assembled, the assembly force follows the characteristic pattern shown in Figure 8.11. This is also described in [8.11] and [8.23]. After a steep rise, the assembly force reaches a peak, falls to a lower level where it remains fairly constant as the lead angle causes the part to deform, and then falls back to zero, once the joint area of the part snaps into place.

Deformation during the assembly of snap-fit joints can be significant. As a result of these deformations during the assembly operation, the geometric relationships change (e.g., the relative angular positions) [8.21, 8.10]. This, however, is not taken into account in the calculation of the assembly forces in the sections below. The local variation of the plane of action and its effect on transverse force during the assembly operation is likewise not taken into account (see Section 5.4).
Figure 8.9  Conversion factor $\eta$ for various coefficients of friction $f$ as a function of the joining (lead-in) angle or the retaining (snap-out) angle [8.11]

Figure 8.10  Forces and moments acting on a snap-fit hook having a retaining angle of $90^\circ$ at the time of release
Loss of Retaining Force

In the case of snap-fit elements that are repeatedly joined and separated, or those that remain under a residual stress, the time-dependence of the material properties should be taken into account. In line with viscoelastic behavior, the strain (deformation) imposed during the assembly operation diminishes only gradually. Test results on separated [8.21] and on assembled snap-fit joints made from POM and PP [8.11] have shown that recovery after release of stress can take as long as 4 to 5 hours. The residual strain found in these cases was in the range of 1 to 3% for disassembly strain values of 8 to 10%.

These residual strain values are reached asymptotically after 5 to 10 assembly or release cycles. Lower assembly related strains lead to lower residual strains. In addition, after a large number of assembly cycles, no further loss of retaining force is observed.

If the snap-fit joint element is deformed enough during assembly resulting in a residual stress, this stress relaxes over time after assembly in line with the relaxation behavior of the material. The residual stress or residual elastic force remaining can be estimated theoretically by linearizing the isochronous stress-strain diagram (see the example calculation in Section 5.3.2).

Figure 8.11  Assembly force over the assembly path for cylindrical snap-fit joints having different size undercuts. Outside part is made of POM (H 2200) with $d_{\text{min}} = 40$ mm; inside part is made of steel [8.18].

Figure 8.12  Relationship between residual strain and number of release cycles for an annular snap-fit joint made of POM having a rigid inner part for different undercut sizes [8.11]
8.1.1 Snap-Fit Beams

8.1.1.1 Types of Snap-Fit Beams

The most common structural element in snap-fit joints is a beam, subject to a bending load, in the form of a cantilever snap-fit beam with a hook. Its useful snap-fit height (momentary interference) can be altered by changing the cross-sectional shape of the beam and, of course, by its effective snap-fit length.

Good utilization of material is reflected in high values for the geometry factor $C$ (see Figure 8.13).

<table>
<thead>
<tr>
<th>Rectangular cross section</th>
<th>Trapezoidal cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = \text{const.}$</td>
<td>$a = \text{const.}$</td>
</tr>
<tr>
<td>$h = \text{const.}$</td>
<td>$b = \text{const.}$</td>
</tr>
<tr>
<td>$C = 0.67$</td>
<td>$h = \text{const.}$</td>
</tr>
<tr>
<td>$h = \text{const.}$</td>
<td>$b \rightarrow b/4$</td>
</tr>
<tr>
<td>$b_x = b \frac{x}{l}$</td>
<td>$C = 0.86$</td>
</tr>
<tr>
<td>$C = 1$</td>
<td>$h \rightarrow a/4$</td>
</tr>
<tr>
<td>$h = \text{const.}$</td>
<td>$b \rightarrow b/4$</td>
</tr>
<tr>
<td>$b = \text{const.}$</td>
<td>$C = 1.28 \frac{a + b}{2b + a}$</td>
</tr>
<tr>
<td>$h_x = h \sqrt{\frac{x}{l}}$</td>
<td></td>
</tr>
<tr>
<td>$C = 1.33$</td>
<td></td>
</tr>
<tr>
<td>$b = \text{const.}$</td>
<td></td>
</tr>
<tr>
<td>$h \rightarrow h/2$</td>
<td></td>
</tr>
<tr>
<td>$C = 1.08$</td>
<td>$a = \text{const.}$</td>
</tr>
<tr>
<td></td>
<td>$b = \text{const.}$</td>
</tr>
<tr>
<td></td>
<td>$h \rightarrow h/2$</td>
</tr>
<tr>
<td></td>
<td>$C = 1.64 \frac{a + b}{2b + a}$</td>
</tr>
</tbody>
</table>

Figure 8.13 Material utilization as reflected by the geometry factor $C$ in snap-fit hooks having different cross-sectional shapes according to [8.5]. The values of $C$ for the trapezoidal cross section apply to the case in which the tensile stress acts in the wide face of the trapezium.

Uniform loading of the material and hence optimum utilization of the material for a cantilever snap beam is achieved by a linear decrease in width or a parabolic decrease in thickness along the length of the beam.
Figures 8.14 to 8.17 indicate some aspects of importance for production in the design of snap-fit hooks. For example, two opposing hooks are more easily produced if they have a cross section in the form of a cylinder segment rather than a rectangular cross section. The simpler production due to the cylindrical shape of the geometric envelope affords substantially lower mold production costs. The production costs for drilling, reaming, and polishing the circular cross section may have a cost ratio of 1:4 compared to those for producing a rectangular cross section by spark erosion and milling [8.7].

By skillful partitioning of the snap features within the mold and the use of shut-offs or piercing cores (see Figure 8.15), snap-fit hooks can be produced without complicated mold actions. When shut-offs are used to produce snap-fit beams and hooks, the designer must allow for the shut-off angle (0.5 to 1°).

In clamshell housing parts, such as those illustrated in Figure 8.16, the undercuts of built-in hooks are most easily molded if the hook faces outward (top) rather than inward (bottom). The maximum stress that occurs when a beam bends is usually at the transition from the snap-fit beam to the molding. *Radii of curvature have to be provided here*, even if this increases mold-making costs. Even a radius of 0.5 mm reduces the peak stress at the transition considerably (see Figures 8.17 and 10.4). Generous radius values are also recommended for segmented annular snap-fit joints (Figure 8.17).

Adequate snap-fit hook height can be achieved by extending the length of the elastic section of a hook (see Figures 8.18 and 8.19).

*Interlocking joints* with a series of joining positions arranged one behind the other allow for assembly at various positions. Figures 8.19 to 8.21 show examples of this concept applied to molded parts.

The concept of an elastic snap-fit beam with a hook and a rigid undercut may also be “reversed” to form the variant of a rigid hook and an elastic beam with an undercut. An example of this is shown in Figure 8.22. Figure 8.23 shows an example of an automobile headlight housing incorporating this concept (see also Figures 8.27 and 7.60).

![Figure 8.14](image_url) Snap-fit hook with circular (a) and rectangular (b) envelope shape and associated details of the injection molds [8.7]
Figure 8.15  Principle of demolding a snap-fit beam and hook without special mold action [8.20]

Figure 8.16  Beam hooks (undercuts) on the core side (bottom) cause higher mold costs than those facing outward (top) [8.13]

Figure 8.17  Rounding-off the segment gap for slotted annular snap-fit joints to reduce peak stress values

Figure 8.18  Principle of extending the length of the elastic (bending) section of a snap-fit hook [8.9]

Figure 8.19  Interlocking joint with saw tooth profile and retaining guard on a clamping ring
Figure 8.20  Interlocking joint capable of adjustment [8.2]

Figure 8.21  Plug housing capable of being fixed sideways in two locking positions

Figure 8.22  Housing cover joint assembled using cantilever beams with undercuts rather than hooks [8.5]

Figure 8.23  Joint composed of a rigid hook and an elastic bracket
8.1.1.2 Snap-Fit Beam Calculations

Permissible Size of Undercut

A snap-fit hook (snap-fit bracket) may be simplified as a bending beam fixed at one end (i.e., a cantilever beam). Calculations can be performed on the basis of classical bending theory*. In the assembly calculations, the beam is theoretically deflected by at least the depth of the undercut. In this rough calculation, the effect of shear stress due to the transverse force is usually neglected because \( l \gg h \). Any deflection of the mating surface is usually estimated or neglected in the classical calculations, although it may be considered in Finite Element Simulations.

The permissible size of the undercut (snap-fit height) for a cantilever snap-fit beam can be determined based on the permissible outer fiber strain \( \varepsilon_{\text{perm}} \) for the material from which the beam will be made.

\[
H_{\text{perm}} = C \frac{l^2}{h} \cdot \varepsilon_{\text{perm}}
\]  

where

- \( C \) = Geometry factor (see Figure 8.13)
- \( \varepsilon_{\text{perm}} \) = Permissible outer fiber strain as an absolute value (m/m)

Guide values for one-shot assembly:
- Semi-crystalline thermoplastics \( \approx 0.9 \varepsilon_Y \)
- Amorphous thermoplastics \( \approx 0.7 \varepsilon_Y \)
- Reinforced thermoplastics \( \approx 0.5 \varepsilon_Y \)

Guide values for frequent assembly:
- Strain at \( \sigma_{0.5\%} \) (see Figure 5.2c).

Snap beams having shapes and cross sections other than those shown in Figure 8.13 cannot usually be analyzed in this way. A method for analysis of beams with more complex cross sections is given in Section 5.4.

Assembly Force and Retaining Force

The assembly force, \( F \), is calculated from the deflection force \( Q \) and the conversion factor \( \eta \).

\[
F = Q \cdot \eta
\]  

where \( \eta \) is obtained from Figure 8.9.

The retaining force is calculated by analogy with the retaining (or return) angle \( \alpha_2 \) (for \( \alpha_2 \geq 90^\circ \) see Figure 8.10). During assembly, plastic deformation may occur so that as a result of changed geometry, the actual retaining force may be smaller than the one calculated [8.11]. Even when the retaining and joining angles are the same, the separating force is a little smaller than the assembly force. This can be attributed to the fact that the bending moment between the planes of action of the actuating forces and the plane of action of the reaction force in the material tends to open the snap-fit connection during assembly.

* Using classical handbook equations or commercially available computer programs such as the SNAPS PC program from BASF or Fittcalc from Ticona.
The *deflection force* is given by

\[ Q = W \frac{E_S \cdot \varepsilon}{l} \]  

(8.4)

where

\[ W = \text{section modulus for a rectangular cross section } (I/c) \text{ where } c = h/z. \]

\[ W = \frac{b \cdot h^2}{6} \]

for a trapezoidal cross section (with tensile stress in the wider face)

\[ W = \frac{h^2}{12} \cdot \frac{a^2 + 4a b + b^2}{2b + a} \]

for other cross sections see Hütte, Dubbel and other reference works.

\[ E_S = \text{secant modulus in MPa for the strain arising associated with the deflection } \]

\[ \varepsilon = \text{strain arising as an absolute value } (m/m) \]

### 8.1.1.3 Additional Functions

**Overstrain Safeguards**

Snap-fit hooks, especially thin fragile ones or those made using brittle materials, must be adequately protected against excessive stress or deflection (see Figures 8.24 and 8.25).

![Figure 8.24](image) Overstrain safeguards for snap-fit hooks [8.19]

![Figure 8.25](image) Snap-fit hooks can be safeguarded against excessive strain or fracture by means of a deflection limit or stop

*(Photograph: Siemens AG, Munich)*
Retaining Guards

In order to prevent inadvertent or unwanted release of a snap-fit joint (see also Figure 8.10) with certainty, the snap-fit hook can be secured after assembly by another element in the structural unit. Figure 8.26 shows one of many possibilities by the example of the base of a coffee machine. The hot plate is inserted into the two parts of the housing where it presses the tab against the snap-fit hook and in this way secures it against release. Additionally, these tabs provide guidance for the assembly of the hot plate and compensate for any tolerance variations.

Opening Aids

An extension of the snap beam (beyond the hook) in the form of a recessed grip is a simple way to facilitate release of a snap-fit joint by hand (see Figure 8.24). Designs, such as the one shown in Figure 8.27, have also proved to be effective. In this case, however, the bending stress has to be absorbed by the very short fillet between the housing and the actual connecting element.

In the locking mechanism shown in Figure 8.28, a spring provides the force required to keep the fulcrum snap-fit in place.

In the case of snap-fit opening aids involving tools, appropriate means of access and gripping must be designed into the parts to be assembled (see, e.g., Figure 8.29).

Energy Storage Devices

Permanent pretensioning is not easily obtained with molded snap-fit connections made of polymeric material due primarily to the limitations of polymeric materials. Therefore, stresses in the joint should be released as much as possible after assembly. When, however, only relatively small amounts of energy are to be stored, e.g., for compensating tolerances or obtaining small prestress, this can be accomplished using pretensioned snap-fit elements made of polymeric materials. Glass-fiber reinforced materials are best for these applications, but unreinforced materials such as POM can also be used (see Figure 8.28). The residual pretensioning can be estimated from the creep modulus $E_c$ (see example calculation in Section 5.3.2).
Figure 8.27  During assembly, mainly the snap-fit bracket is deformed, while during separation, only the short fillet is deformed.

Figure 8.28  Locking mechanism for a housing cover (Photograph: Siemens AG, Munich)

Figure 8.29  Opening a pipe clamp by means of a screwdriver

Seals

Reliable sealing of two components joined by snap-fit beams can be achieved only if a sufficient number of snap-fit beams is provided and if the pressure of the elastic seal is accomplished by tensile stress (not bending stress) in the beam (see Figure 8.30).

Figure 8.30  Principle for designing an elastic seal